Sandbox modeling of the shallow tunnel face collapse

Pavlos Vardoulakis*, Maria Stavropoulou**, George Exadaktylos*

Summary
The objective of the present work is the experimental, analytical and numerical investigation of the face instability mechanism of shallow circular cylindrical tunnels that are excavated in frictional geomaterials characterized by small cohesion. For the experimental investigation a small scale sandbox model has been constructed. Subsequently, experiments have been carried out at 1g using the same dry sand and changing each time the overburden height above the crown of the tunnel. In a first attempt to derive a simple analytical equation relating the maximum subsidence occurring at the various stratigraphic horizons above the tunnel imparted by the horizontal tunnel face retreat, the model test results were analyzed utilizing the Dimensional Analysis and Displacement Diffusion theories. The resulting analytical equation, that is remarkably simple, could be useful for the design and construction stages of shallow tunnels were the subsidence and face movements or face pressure exerted by the TBM are monitored, as well as for the validation of numerical codes. The experimental results were also compared with numerical results obtained by virtue of the FLAC3D code. It was found that the best agreement of experimental and numerical data is achieved for elastic modulus of the dry sand equal to 130 MPa that was also inferred independently from triaxial compression test results.

Keywords: shallow tunnels, face instability, physical modeling, sandbox testing, dimensional analysis, displacement diffusion, tunnel collapse.

1. Introduction
Tunneling projects are technologically challenging projects. The construction of shallow road, metro or railway tunnels under urban areas around the world has considerably increased during the past two decades. The reasons for going underground are many; for example to increase the speed of transportation of the citizens, to cope with increasing number of new inhabitants in megacities, to considerably decrease the pollution from traffic, and to free the ground space for the inhabitants to name a few. For example Barcelona’s population increased from 3 million to 4 million during the last years with a consequence 9 more metro tunnels plus 2 hydrotunnels to be currently under construction or planned to be constructed in the next few years), to free ground space from traffic etc. The diameter of Tunnel Boring Machines (TBM’s) and particularly slurry shields, mixshields and Earth Pressure Balanced (EPB) machines that are most usually used for the driving of shallow tunnels in soft ground is continuously increasing; e.g. 14.87 m slurry shield in Groene Hart, Netherlands, 14.14 m slurry shield in Tokio Bay Aquiline, Japan, 14.2 m diameter mixshields in Hamburg, Germany and Moscow Lefortovo, Russia, 13.21 m diameter mixshield in Kuala Lumpur, Malasia etc., 12 m diameter EPB’s are currently used for the construction of L9 metro line in Barcelona, 15 m diameter EPB machine is also currently used for the construction of the Madrid metro extension etc. It is envisaged that in the immediate future TBMs with a diameter of 20 m or more will be used in tunneling.

In many cases a TBM may encounter a loose geological material (i.e. soil or heavily fractured rock), therefore a pressure should be applied on the tunnel’s face to prevent its instability or excessive ground surface subsidence. This pressure could be originated from compressed air, slurry mixture (slurry shields principle) or the excavated geomaterial itself mixed with foams and additives to give to the material in the excavation chamber the desired rheological properties (EPB working principle). There is a long list of shallow tunnel collapses in urban areas mostly occured using the New Austrian Tunneling Method (NATM) or more correctly “open front” or “cyclical tunnel excavation” in loose ground; one collapse every 10 km on average during the construction of Hannover-Würzburg line of German Rail, many collapses
during the construction of the Heathrow tunnel, two collapses during construction of Athens metro tunnel etc.

The assessment of the safety of shallow underground excavations in soft ground, both for tunnel construction and mining, usually requires solutions of two separate predictive problems. Firstly, it is necessary to determine the stability of the excavation, for the safety of those at the surface and underground. Secondly, to prevent damage to surface or subsurface structures, it is also necessary to determine the pattern of ground deformations that will result from the construction works. This paper deals with the second of these problems; the aim is to understand how the release of pressure applied to the tunnel face through a controlled retreat of the rigid support of the face, is transferred to the higher stratigraphic layers above the crown of the tunnel and finally to the ground surface. For this purpose a special small-scale sandbox device has been created and then several experiments with dry sand were carried-out at three overburden height to tunnel diameter ratios. Subsequently, fundamental principles of Dimensional Analysis and Displacement Diffusion Theories have been applied in order to derive a simple dimensionless relationship for the dependence of the maximum subsidence of any stratigraphic layer above the shallow tunnel on the horizontal displacement of the rigid tunnel support. It is shown during the process of the derivation of the analytical formula that the subsidence of the dry sand above the retreating vertical tunnel face is a diffusive process. Finally, preliminary numerical simulations with FLAC3D [ITASCA, 2002] were executed and the first results are found in close agreement with the experiments.

2. Experimental Set-up

2.1. Physical and mechanical parameters of the model material

A calcareous sand with angular grains having a unit weight of 1.514 g/cm³ in dry state, was used in the sandbox tests. The mineralogical composition of the sand that was sampled from Kalathas beach close to the city of Chania, is displayed in Table I. The grain size distribution of the sand obtained from sieving three different samples, is presented in Figure 1. The shape of the distribution is characteristic of a relatively well-graded sand.

Furthermore, the percent passing by weight, N, is given in terms of the grain size, d, expressed in μm, by the following “logistic” function

\[
N \equiv \frac{100}{1 + \exp \left[ -\frac{d - 290.4}{44.6} \right]} \tag{1}
\]

Regarding the strength of the sand, as it is illustrated in Figure 2 the peak friction angle of this type of sand in dry condition was estimated from triaxial compression tests by employing the Coulomb-Mohr failure criterion to be \( \phi_p \equiv 37^\circ \).

Tab. I – Mineralogical composition of the sand.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Chemical composition</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartzite</td>
<td>SiO₂</td>
<td>50.1</td>
</tr>
<tr>
<td>Calcite</td>
<td>CaCO₃</td>
<td>38.8</td>
</tr>
<tr>
<td>Magnesium Calcite</td>
<td>(Mg₁₂Ca₈7) (CO₃)₈</td>
<td>8.9</td>
</tr>
<tr>
<td>Dolomite</td>
<td>CaMg(CO₃)₂</td>
<td>2.3</td>
</tr>
</tbody>
</table>
The modulus of elasticity of the dry sand was also evaluated from the triaxial compression tests by fitting straight lines to the unloading-reloading loops performed in each test. As it may be seen from the diagram of Figure 3 the calculated elastic modulus of the sand clearly displays a more or less linear pressure-dependency. This line has been extrapolated for pressures lower than 200 kPa down to few kPa’s which is the magnitude of mean pressure occurring at the tunnel crown level of the sandbox experiments for H/D=0.5, 1 and 2. From the same figure it may be inferred with some caution that the elastic modulus, E, of the sand at sufficient small mean normal stress, $p$, is $E \approx 130$ MPa, $(p/E) \to 0$. It is worth mentioning, that this order of magnitude of the elastic modulus of sand is in accordance with numerical simulations that predict the same more or less settlements above the tunnel that will be presented in Section 4 of this paper.

Finally, the dilatancy angle of the sand, which is herein denoted as usual by the lowercase Greek letter \( \psi \), was estimated from a series of direct shear tests on more or less loosely pluviated dry sand with a unit weight of $\gamma_d=1.514 \text{g/cm}^3$, by measuring the vertical and horizontal displacements. As it is shown

---

**Fig. 2** – Mohr envelopes of sand obtained from triaxial compression tests and bestfitted Coulomb-Mohr linear failure criterion.

**Fig. 2** – Inviluppo di rottura di Mohr per la sabbia ottenuto con prove di compressione triassiale e interpolazione con la retta di Mohr-Coulomb.

**Fig. 3** – Linear dependence of the elastic modulus of the dry sand on mean normal stress inferred from the triaxial compression tests. The variables of the best fitted linear eqn \( y \) and \( x \) refer to \( E \) and \( p \), respectively.

**Fig. 3** – Andamento lineare del modulo elastico della sabbia asciutta in funzione della pressione normale media dedotto da prove di compressione triassiale. Le variabili della retta di interpolazione, \( y \) e \( x \), si riferiscono rispettivamente a \( E \) e \( p \).

---

**Fig. 4** – A thin slice of thickness $d$ of a sheared granular material that exhibits dilatancy (increase of volume and porosity), in which $\dot{u}_v=\partial u_v$, $\dot{u}_h=\partial u_h$ for time-independent deformations.

**Fig. 4** – Un sottile strato di materiale granulare, di spessore $d$, sottoposto a deformazione di taglio e manifestante comportamento dilatante (aumento di volume e porosità), con $\dot{u}_v=\partial u_v$, $\dot{u}_h=\partial u_h$ per spostamenti indipendenti dal tempo.
in Figure 4 the dilatancy angle can be estimated from the following formula

$$\psi = \arctan \left( \frac{\partial u_v}{\partial u_h} \right)$$  \hspace{1cm} (2)

where $u_v$, $u_h$ denote vertical and horizontal displacements.

The dependence of the vertical displacement on the horizontal displacement during the three shear tests are illustrated in Figure 5. Best-fitted polynomial curves shown in the same figure are subsequently used for the computation of the slope $\partial u_v/\partial u_h$ and then through Equation (2) for the construction of the diagram illustrated in Figure 6 that refers to the variation of the tangent of the mobilized dilatancy angle (dilatancy coefficient) with the horizontal displacement for the three tests at hand. From this diagram it was inferred that the peak dilatancy angles of the sand are $\psi_p \cong 6.4^\circ$, $6.3^\circ$ and $5^\circ$, for the direct shear tests corresponding to an applied normal stresses of 30, 46 and 60 kPa, respectively. In the same figure the evolution of the mobilized friction coefficient of the dry sand with the horizontal displacement for the three tests at hand, is also displayed.

It was found that for this particular quartzitic sand the following approximate empirical relation holds true

$$\psi_p = \phi_p - 31^\circ$$  \hspace{1cm} (3)
It may be shown that the two hypotheses of Taylor's theory [TAYLOR, 1948] lead to the result that a granular material that is characterized by internal friction and dilatancy, may be considered to behave equivalently, from an energetic point of view, with a material that obeys Coulomb’s friction law and deforms in an isochoric manner. The internal friction angle $\phi_{eq}$ of this equivalent material is given by the following equation

$$\tan \phi_{eq} = \tan \phi - \tan \psi$$

From the direct shear tests we may deduce the following values of strength parameters of the loose Kalathas sand: (a) a peak friction angle $\phi_p = 37^\circ$ that is in accordance with that obtained independently from the triaxial compression tests, (b) a peak dilatancy angle $\psi_p = 6^\circ$, (c) a critical state friction angle $\phi_c = 35^\circ$, and (d) a Taylor’s equivalent friction angle $\phi_{eq} = 33^\circ$.

2.2. Description of the sandbox device

A special sandbox device was created to simulate TBM-driven shallow tunnels in a frictional ground characterized by a small cohesion (see Appendix A). As it is illustrated in Figure 7, only one-half of the tunnel cross-section was simulated using this device, since the problem under investigation is symmetrical in the vertical plane passing through the tunnel axis. As may be seen in the same figure this design permits also the observations and measurements of displacements of the sand both along the longitudinal axial of the tunnel and on a vertical plane normal to the tunnel axis. The dimensions of the sandbox are 54x152x60 cm whereas the casing material was waterproof wood in order to be able to perform experiments with wet sand, as well.

After the construction of the wooden box, double-sheeted transparent glasses were fixed on the three sides of it (two lateral sides and in the front) to facilitate observations during the experiments. The inner sheet is a crystalline glass and the outer is a Plexiglas with thicknesses of 0.5 and 1 cm, respectively. The insertion of the inner crystalline glass was done in order on one hand to reduce the interfacial friction between the transparent walls with the quartzitic sand, and on the other hand to avoid the scratching of the surface of the Plexiglas during each sand pluviation. After fixing the glasses on the box, a semi-circular hole with a diameter of D=7 cm was opened on the two lateral glass-walls and at the contacts of these lateral walls with the front glass wall (Fig. 7). Through that hole a tube was subsequently fixed that was also made from Plexiglas. In that tube a piston was inserted with a rigid face which could be pushed inwards or pulled outwards with help of a screw (see Fig. 8). This experimental configuration corresponds to a tunnel excavation with a shield machine immediately followed by a lining of concrete rings, in such a manner that a fully rigidly supported tunnel periphery and face to occur. Face instability may be simulated by pulling outwards the piston thus imposing a certain movement of the tunnel face (i.e. extrusion of the tunnel face).
Initially, the movable lateral box shown in Figures 7 and 8, was put at a certain fixed distance from the tunnel and fixed at that position. In all the experiments this distance was set equal to 1.5xD. In each experiment, the sand was carefully pluviated in the empty space formed between the wall of the lateral wooden box and the tunnel, with the use of a funnel to achieve a relatively loose packing of the sand.

The sand layering procedure was done up to certain predefined height in an stepwise manner. At certain stratigraphic horizons the pluviation of the sand stopped and a horizontal surface was achieved by removing the excess sand with the use of a vacuum pump. At this point a thin layer of black-colored sand was pluviated in order to facilitate the observations of the vertical displacements with the use of a high-definition digital camera. The typical distance between two adjacent black layers was set equal to 1.5 to 2 cm (Fig. 8).

Model material selection and model scaling with respect to the geostatic stress field based on geometrical similitude arguments are presented in Appendix A.

2.3. Representative sandbox experimental results on the “chimney effect”

A total of nine experiments were carried-out with height over tunnel diameter ratios (H/D) equal to 0.5, 1, and 2, respectively. Once the desired height was achieved, the incremental pull-out of the tunnel face piston was commenced (Fig. 9). As it is shown in Figures 9 (a-d), the backwards movement of the piston was made at a constant speed so that the successive stages of the localization and forma-
tion of the “chimney” in the front and above the face of the tunnel could be easily observed.

At certain stages of the experiment photographs where taken that were later analyzed in order to find a correlation between tunnel-face displacement and surface subsidence in the front and above the tunnel. The processing of the photographs and the measurements on them were made with the use of an autodesk design application as it is shown in Figure 10. The accuracy of the measurements of vertical displacements of the sand, as well of the horizontal displacement of the face using this technique was of the order of the 1÷2 mm. The collected data were subsequently stored in spreadsheets in order to find the correlation between the various dimensionless parameters of the problem at hand.

3. Dimensional Analysis of the experimental data

3.1. Basic equations

It is assumed that the maximum vertical subsidence \( u_y \) of the various sand layers is a function of \( y \) which is the distance of this layer from the free surface, the horizontal displacement of the tunnel face support \( u_x \), the geometrical characteristics of the given problem shown in Figure 11, namely the diameter \( D \) of the tunnel, and the height of the overburden \( H \), and finally from the properties of the sand, such as the elastic modulus, Poisson’s ratio \( \nu \), unit weight, friction and dilatancy angles. That is to say,

\[
U_y = f_0(y, u_x, H, D, \phi, E, \gamma, \ldots)
\]  

(5)

The above equation may be further simplified by assuming isochoric deformations \( \psi = 0 \) by adopting Taylor’s equivalent sand concept with equivalent internal friction angle \( \phi_{eq} \)

\[
U_y = f_1(y, u_x, H, D, \phi_{eq}, E, \gamma, \ldots)
\]  

(6)

The dimensions of the problem are length and force. Buckingham’s theorem which is used in dimensional analysis, states that \([HORNING, 2006]\): if \( N_2 \) is the total number of the variables of the problem and that \( N_3 \) is the number of the standard dimensions that are needed to express the dimensions of all the variables, then the number of the independent dimensionless variables that describe the problem \( N_1 \) is given by the relationship

\[
N_1 = N_2 - N_3
\]  

(7)

Since in our case from Eq. (6) \( N_2 = 9 \) and \( N_3 = 2 \), then according to Eq. (7) \( N_1 = 7 \). Therefore, apply-
ing Buckingham’s theory Eq. (6) may be written in the following dimensionless form

\[ \frac{u_y}{D} \approx \int_2 \left( \frac{y}{D}, \frac{u_x}{D}, \frac{H}{D}, \frac{E}{\gamma D}, \frac{\phi_{eq}}{y} \right) \]  

(8)

3.2. Displacement diffusion in the tunnel-head problem

We assume that in front of the tunnel head an almost rigid wedge is formed that transfers the vertical movement at \( y = 0 \) to the tunnel face horizontal movement (Figs. 1, 2),

\[ U_{s0} = \lambda u_{y0} \quad (0 < \lambda \leq 1) \]  

(9)

We also assume that the subsidence in the chimney is diffusing upwards, as is the case in the so-called trap-door problem [PAPAMICHTOS et al., 2001; VARDOUNAKIS et al., 2004]. For simplicity we consider here the “plane” problem, in order to justify the expected relationship between vertical subsidence inside the chimney and horizontal yield of the tunnel face. Within the theory of upwards diffusion of subsidence the vertical coordinate plays here the role of the time-like independent variable, and the subsidence is obeying the diffusion equation

\[ \frac{\partial u_y}{\partial y} = \epsilon \frac{\partial^2 u_y}{\partial x^2} \]  

(10)

The diffusivity coefficient \( \epsilon \) has the dimensions of length. We normalize the independent variables \( x, y \) by the width \( 2R \) of the chimney (Fig. 12) that in turn is set approximately equal to a percentage of the tunnel diameter,

\[ x^* = \frac{x}{R} \quad ; \quad y^* = \frac{y}{R} \]  

(11)

and with that Eq. (10) becomes,

\[ \frac{\partial u_y}{\partial y} = R \epsilon \frac{\partial^2 u_y}{\partial x^2} \]  

(12)

At any given displacement of the tunnel face \( u_{s0} \), we assume that at level \( y = H \) the subsidence \( u_{y0} \) is uniform along the Ox-direction. We then normalize the subsidence by this value,

\[ \frac{u_y}{u_{y0}} = \frac{u_y}{u_{y0}} \]  

(13)

For simplicity we will drop in the following the superimposed asterisks and assume that all quantities are dimensionless. Thus we introduce here, as in the trap-door problem, the following “initial” condition,

\[ u_y(x,y) = u_{y0} \quad \text{for} \quad -2 \leq x \leq 0 \]  

(14)

As boundary conditions we assume vanishing subsidence at the chimney walls,

\[ u_y(0,y) = u_y(-2,y) = 0 \quad \text{for} \quad y > 0 \]  

(15)

Equation (12) with imposed conditions (14) and (15) constitutes a classical diffusion problem, that is treated in any textbook of Soil Mechanics, within the theory of consolidation [TAYLOR, 1948]. This theory allows us to propose that the average subsidence at elevation \( 0 < y \) is given by the approximate formula,

\[ U = 1 - 2 \sqrt{\frac{T}{\pi}} \]  

(16)

In this expression \( T \) is the “time factor”,

\[ T = \frac{\bar{x}^2}{\epsilon} \]  

(17)

and

\[ U = \frac{u_y}{u_{y0}} ; \quad \bar{u}(y) = \frac{1}{2R} \int_0^{2R} u_y(x,y) \, dx \]  

(18)

Thus,

\[ \bar{u}(y) = \frac{1}{\lambda} \frac{u_{s0}}{D} \left( 1 - 4 \sqrt{\frac{\pi}{\epsilon \sigma^2 D}} \sqrt{\frac{y}{D}} \right) \]  

(19)

This qualitative analysis suggests to search for a dependency of vertical subsidence on the square root of its distance from the tunnel crown. Also, it turns out that the hypothesis that large-scale subsidence over a yielding underground geostructure,
such as the support on the tunnel face, is a stochastic Markov process is valid [Litwiniszyn, 1974; Dimova, 1990]. In simple terms this hypothesis implies that the gravity driven displacement of a particle in vertical direction causes the movement of a number of particles lying above it, thus resulting in an upwards convection and lateral spread of the vertical displacement horizontally.

3.3. Derivation of the dimensionless analytical equation among subsidence and horizontal face retreat

According to the above diffusion Equation (19) and Equation (8) the following equation should hold true in our experiments assuming a constant Poisson’s ratio \( \nu \cong 1/3 \), \( E/\gamma D = 122700 \) that holds true for sufficient small mean normal stress such as this occurring in the small scale sandbox tests, and for \( \phi_{eq} \cong 33^\circ \), that is

\[
\frac{u_y}{D} = b \left( \frac{u_x}{D}, \frac{H}{D} \right) + a \left( \frac{u_x}{D}, \frac{H}{D} \right) \cdot \sqrt{\frac{\gamma}{D}} \quad (20)
\]

where as is indicated in the above expression parameters \( a \) and \( b \) are functions of the dimensionless horizontal face displacement and the dimensionless height of the overburden. From Figures 13(a-c) it may be observed that the subsidence for the relative overburden heights \( H/D = 0.5, 1 \) and 2 indeed obeys the diffusion solution (20) since the vertical displacement depends on the square root of the depth in a linear fashion. Moreover, the parameter \( a \) which is the slope of the linear best-fit curves passing through the pairs of dimensionless subsidence and square root of the dimensionless depth of a layer, could be considered in a first approximation to be independent of the relative face displacement as it may be seen from Figure 14. So, by taking average values of the slopes at each burial ratio of the tunnel \( H/D \) it may be found that the average value of the slope \( a \) has the following approximate dependence on the relative depth of the layer, i.e.

\[
a \cong 0.14 \left( \frac{H}{D} \right) \quad (21)
\]

Based on the above considerations, the general dimensionless equation for the subsidence at any stratigraphic horizon could be derived if the relationship \( b = b(u_x/D, H/D) \) is found.

This task is accomplished by plotting the intercept parameter \( b \) w.r.t. the relative face displacement as it is shown in Figure 15. From the linear regression analysis of the three set of test data referring to \( H/D=0.5, 1 \) and 2 the following approximate relation for parameter \( b \) is obtained

\[
b \cong -0.14 \left( \frac{H}{D} \right)^{3/2} + 1.55 \frac{u_x}{D} \quad (22)
\]

It should be mentioned here that the initial best-fitted line for the data corresponding to \( H/D=1 \) gave a larger slope than 1.55 which was found for the other two cases, i.e. \( H/D=0.5 \) and \( H/D=1 \). For this purpose in a second attempt, we prescribed the intercept of the line to be approximately equal to the first term of the r.h.s. of Equation (22) and then obtained the curve shown in Figure 13. In the same figure it may be seen that one point corresponding to \( u_x/D = 0.05 \) lies considerably far from the theoretical line; this may be attributed to a measurement error.

Finally, the single approximate equation the gives the relative subsidence at any stratigraphic level is found by combining Equations (20-22)

\[
\frac{u_y}{D} = \begin{cases} -0.14 \left( \frac{H}{D} \right) \left[ \sqrt{\frac{H}{D}} - \frac{\sqrt{\gamma}}{D} \right] + 1.55 \frac{u_x}{D} \quad & \text{if } \frac{u_x}{D} > 0 \\ 0 \quad & \text{if } \frac{u_x}{D} \leq 0 \end{cases} \quad (23)
\]

Equation (23) depicts that the maximum dimensionless subsidence at the level of the crown of the tunnel, i.e. for \( y = H \), is independent on the relative height of the overburden \( H/D \); this observation comes from the sandbox experiments, and this is the reason for the appearance of the first term in the r.h.s. of Equation (23). Moreover, it depends linearly on the dimensionless retreat displacement of the tunnel support. From a back analysis that was conducted afterwards for \( y=H \) and \( y=0 \) and presented in Figures 16 and 17, respectively, it was found that even the simple Equation (23) can describe fairly accurately the maximum subsidence inside the zone of the localized deformation that takes place during a relaxation of the tunnel face.

4. Comparison of numerical results with the experimental results

A numerical investigation of the sandbox model was performed for the comparison with the analogue model results. For this purpose the 3D large-strain explicit finite differences numerical code FLAC3D by ITASCA [2002] was employed. First, the geometry of the sandbox was properly depicted for the numerical model. In order to keep in the numerical model the same analogies with the model tests, the tunnel diameter was kept constant \( D=7 \) m, the total length of the reinforced tunnel was \( L=41 \) m, the width of the lining is 0.5 m,
The horizontal distance of the vertical wall from the tunnel axis was set equal to 30 m, the distance of the right-wall from the tunnel-face was set equal to 10 m. As observed from experiments, the left and right sidewalls (see Fig. 7) have no effect on the chimney formation since they are far enough apart. Therefore, the discretization of the numerical model includes only the sand, the rigid lining along the tunnel, the movable rigid face support, and the supporting structure. After the geometry description, the material properties for the sand, the tunnel lining and face, as well as the box are

Fig. 13 – Dependency of the dimensionless vertical displacement on the square root of the dimensionless elevation position for various horizontal displacements in the cases of (a) $H/D=0.5$, (b) $H/D=1$ and (c) $H/D=2$. The dependent and independent variables of the best fitted linear eqns namely $y$ and $x$, refer to $u_y/D$ and $y/D$, respectively.

Fig. 13 – Andamento dello spostamento verticale (adimensionale) in funzione della radice quadrata dell’elevazione (adimensionale) per vari spostamenti orizzontali nei casi (a) $H/D=0.5$, (b) $H/D=1$ and (c) $H/D=2$. Le variabili dipendenti e indipendenti della retta di interpolazione, $y$ and $x$, si riferiscono rispettivamente a $u_y/D$ e $y/D$. 

the horizontal distance of the vertical wall from the tunnel axis was set equal to 30 m, the distance of the right-wall from the tunnel-face was set equal to 10 m. As observed from experiments, the left and right sidewalls (see Fig. 7) have no effect on the chimney formation since they are far enough apart. Therefore, the discretization of the numerical model includes only the sand, the rigid lining along the tunnel, the movable rigid face support, and the supporting structure. After the geometry description, the material properties for the sand, the tunnel lining and face, as well as the box are

RIVISTA ITALIANA DI GEOTECNICA
The dry sand with unit weight $\gamma_d = 1.514 \text{g/cm}^3$ is modeled as an elastic-perfectly plastic Coulombic, purely frictional material with a non-associative flow rule, and with a constant friction and dilatancy angles of $\psi = 37^\circ$ and $\phi = 6^\circ$, respectively. The Poisson’s ratio for sand was put equal to $\nu = 0.3$ and its elastic modulus $E = 130 \text{MPa}$ as was inferred from the triaxial cell tests (see Fig. 3). For the better simulation of the experiment, the concrete lining of the tunnel and the TBM head on the tunnel face were simulated as rigid bodies. Roller boundary conditions that keep horizontal displacements equal to zero are prescribed along the four vertical boundary walls that hold the sandpack. The top surface is stress-free whereas the nodes of the bottom surface of the model are all fixed in both dimensions.

Each simulation was conducted in two phases: an initial analysis to establish initial equilibrium under the influence of gravitational body forces, and boundary restraints, followed by a phase during which the displacement velocity is imposed on the face support in a stepwise manner in small time increments. The grid employed in the numerical model is illustrated in Figure 18 for the case of H/D=2.

The next stage of the simulation was the backward (relaxing) displacement of the tunnel face $u_x$. Following that, the code was left to reach equilibrium again. After the solution of the model for each H/D ratio it was made clear that the failure zone in front and above the tunnel resemble to the ones that
where observed during the experiments (i.e. Figs. 19 (a-c)).

Figure 16 illustrates the preliminary comparison of the prediction of the maximum dimensionless subsidence at the tunnel crown level, based on the analytical Equation (23), on the sandbox experimental results, and on a FLAC$^{3D}$ run for one value of the relative tunnel face retreat. From this figure it may be observed that all experimental points fall onto the same curve irrespectively from the H/D ratio; also the predictions of the analytical equation and of the numerical code are in close agreement with experimental data.

It should be noticed that the possible pathological dependence of the numerical results on the mesh discretization was not checked in this first attempt. This will be done in a future work. However, there was made a comparison with Z-SOIL code [SARF, 2007] also with a coarse mesh and it was found that the numerical results regarding localization of deformation and subsidence pattern are comparable with those obtained with FLAC$^{3D}$.

Concluding remarks

The objective of this work was to study the transfer of displacements all the way up to the free ground surface imparted on a frictional geomaterial characterized by small cohesion from a relieving
tunnel face. This was achieved by employing a small scale sandbox device that was constructed specially for this purpose. The main conclusions derived from this study may be summarized as follows:

the 1g experiments with the new sandbox apparatus displayed similar shapes of the failure zone above a shallow tunnel as they are observed in real life.

The derived analytical equation based on fundamental principles of Dimensional Analysis and Displacement Diffusion theories, gives fairly good predictions of the maximum subsidence at each stratigraphic level in front and above the tunnel face up to the free surface, due to face pressure relaxation.

FLAC3D captures both the shape of the localization zone and subsidence pattern in a fairly good fashion. More work should be done to validate properly this or other numerical code regarding both the shape of the failure region above the tunnel and the subsidence pattern.

More experiments with the same apparatus are planned with wet sand and other types of geomaterials (e.g. clays) or even inhomogeneous material consisting from alternating layers of clay and sand.

Acknowledgements

The authors would like to thank the financial support from the EC 6th Framework Project TUN-CONSTRUCT (Technology Innovation in Underground Construction) with Contract Number: NMP2-CT-2005-011817.
Eqs. (A.2) and (A.3) mean that the deformational and cohesive properties of the model material must be also down-scaled to the same extent as the geometric dimensions. In order to satisfy these model laws, one must use as model material a material that has a very low elasticity and strength. This observation suggests to select cohesionless sand as model material (Hubbert, 1937; Carter et al., 1981) at least for the satisfaction of the low strength criterion.

Similarly, for the vertical geostatic stress \( \sigma_y = \gamma y \)

\[
\frac{\sigma_y}{\gamma D_p} \approx \frac{\sigma_y}{\gamma M} \approx 264 \quad (A.4)
\]

This means that the 1g sandbox tests simulate large enough stresses occurring in real-life and no centrifuge tests are necessary to artificially increase the gravity.

### Modellazione in scala ridotta del collasso del fronte di scavo di gallerie superficiali

#### Sommario
Il presente lavoro ha come obiettivo lo studio per via sperimentale, numerica e analitica del meccanismo di instabilità del fronte di scavo di gallerie cilindriche a sezione circolare scavate in geomateriali attritivi caratterizzati da debole coesione. Un modello fisico a scala ridotta è stato realizzato appositamente per lo studio sperimentale. In seguito, le prove sono state effettuate in condizioni di gravità normale, sulla stessa sabbia asciutta e cambiando ad ogni prova l'altezza del riempimento al di sopra del coronamento della galleria. Per un primo tentativo di derivazione di una semplice relazione analitica descrittiva dei valori massimi della subsidenza in corrispondenza di vari orizzonti stratigrafici al disopra della galleria e indotti dall'arretramento del fronte di scavo, i risultati delle prove su modello fisico sono stati analizzati con l'ausilio delle teorie dell'analisi dimensionale e della diffusione degli spostamenti. La relazione analitica che ne risulta, particolarmente semplice, si propone come strumento di ausilio nelle fasi di progetto ed esecuzione di gallerie superficiali nei casi in cui siano monitorati la subsidenza e i movimenti del fronte di scavo o la pressione sul fronte esercitata dalla fresa meccanica (TBM), così come ai fini della validazione di codici di calcolo. I risultati delle prove sperimentali sono inoltre confrontati con i risultati ottenuti tramite il codice FLAC3D. Ne risulta che il migliore accordo tra dati numerici e misure sperimentali si ottiene in corrispondenza di un valore del modulo elastico della sabbia asciutta pari a 130 Mpa, dedotto inoltre indipendentemente dai risultati delle prove di compressione triassiale.

#### Parole chiave: gallerie superficiali, instabilità del fronte di scavo, modellazione fisica in scala ridotta, analisi dimensionale, diffusione degli spostamenti, collasso di gallerie.

### Appendix A

The geometric dimension (i.e. the diameters of the tunnel) \( D_p \) of the prototype \( P \) is scaled down in a small-scale lab model test \( M \) to the value \( D_M \). For example 1\( m \) in the model may represent 2\( m \) in full-scale tunneling. That is to say our case of \( D_M=0.07m \) of the model corresponds to \( D_p=14m \) tunnel diameter in full-scale, which is not an unrealistic value. In that case the geometric scale is

\[
\frac{D_M}{D_p} = \frac{1}{200} \quad (A.1)
\]

Eq. (8) is the analytic expression of the dependence of the subsidence above tunnel, Similitude requirements imply that for \( y_p=2g/cm^3 \) and \( y_M = 1.514g/cm^3 \)

\[
\frac{E}{\gamma D_p} = \frac{E}{\gamma D_M} \Rightarrow \frac{E_M}{E_p} = \frac{y_M}{y_p} \frac{D_M}{D_p} \approx 0.0038 \quad (A.2)
\]

and for a cohesive Coulombic material with cohesion \( c \)

\[
\frac{c}{\gamma D_p} = \frac{c}{\gamma D_M} \Rightarrow \frac{c_M}{c_p} = \frac{y_M}{y_p} \frac{D_M}{D_p} \approx 0.0038 \quad (A.3)
\]