Equivalent loading for seismic response analysis of Newmark’s block

Paolo Carrubba,* Paolo Pavanello**

Summary

The aim of the study was to evaluate seismic displacements for Newmark’s block, provided with Coulomb friction and resting over horizontal or inclined planes, from a collection of Italian seismic records. This subject concerns the seismic design of geotechnical structures of the sliding type, such as quiescent landslides, earth dams and gravity works. Depending on the plane sloping and block friction, several numerical integrations of the equation of the motion were performed, in order to outline the maximum block displacements. The records of 38 Italian earthquakes were employed in these analyses, so that a wide set of data related to seismic events of magnitude ranging between $3.7 < M_L < 6.5$ was obtained. The analyses were performed for both horizontal acceleration alone and for simultaneous horizontal and vertical accelerations, acting at the base of the model. Afterwards, the block displacements were also evaluated for sinusoidal base motions, varying in amplitude, duration and frequency. These latter displacements were used for normalization purposes: by comparing the seismic and the sinusoidal displacements, an equivalence criterion was established for the selected 38 Italian seismic events. Therefore, the seismic displacement of Newmark’s block, resting on an inclined plane, were predicted by using three kinematic parameters: $PGA$, $PGV$ and $t_{sg}$, which are the peak ground acceleration, the peak ground velocity and the significant duration of the earthquake.

Introduction

It is well known that the free-field seismic motion of the earth’s crust is characterised by a large number of parameters, which in turn control the processes of storing, releasing and transmitting the seismic energy from the focus towards the earth’s surface. Interferences between seismic waves and geomorphology, as well as induced modification in ground motion by soil stratigraphy, are known to be highly influential in selecting the design earthquake at a site of interest. Finally, the interaction between ground motion and a civil structure is the final stage in quantifying seismic risk in civil engineering.

To support engineering decisions in seismic areas, simplified approaches have been frequently proposed by many researches, to try to solve specific geotechnical problems. For example, the prediction of post-seismic displacements of dams, slopes and gravity walls, has been carried out by simplified approaches [Newmark 1965; Franklin and Chang, 1977; Makdissi and Seed, 1978; Richards and Elms, 1979; Whitman and Liao, 1984; Siddharthan and Norris, 1991; Yegian et al., 1991, Caltabiano et al., 1999].

From this standpoint, the significant number of uniform cycles of loading [Newmark, 1965; Cau and Bathurst, 1996; Bommer et al. 2006] can be considered to be a potentially useful tool for quantifying the effect of any time history on a complex geotechnical system. Starting from the considerations outlined by Sarma [1975] and by Yegian et al. [1991] a numerical approach was followed in this work to evaluate the displacements of a Newmark’s block over horizontal or inclined plane and subjected to a base motion. The aim of the study was to predict the seismic displacements of geotechnical structures, of the sliding type, on the basis of a few simple kinematic parameters.

A selection of 38 Italian seismic records was taken from the SISMA (http://sisma.dsg.uniroma1.it/) and the ITACA (http://itaca.mi.ingv.it/ItacaNet/) databases. Depending on the plane sloping and block friction, several numerical integrations of the equation of the motion were performed, in order to obtain the maximum block displacements under seismic events of magnitude ranging between $3.7 < M_L < 6.5$. 

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Displacement analysis was also carried out for sinusoidal loadings, varying in amplitude, duration and frequency. These latter displacements were used for normalization purposes: by comparing the seismic and the sinusoidal displacements, an equivalence criterion was established for the Italian seismic events.

**Selected seismic motions and their characterisation**

The seismic ground motions employed in the analysis mainly came from the SISMA database (Scassera et al., 2008), which is the result of a joint project between the University “La Sapienza” (Rome, Italy) and the University of California Los Angeles (USA). Additional information was supplied by the ITACA database, collected by the Italian Istituto Nazionale di Geofisica e Vulcanologia.

In this context, 38 records of Italian earthquakes were selected according to the synthesis of Tab. I. The distances between the recording station and the epicentre range from 1 km to 45 km and the Richter magnitude range between 3.7 < $M_R$ < 6.5. The soils beneath the recording stations were divided according to Eurocode EC8. The A Class is representative of very rigid soils in which shear wave velocity is $V_{s,30}$ > 800 m/s. The B Class accounts for dense soil deposits with 360 m/s < $V_{s,30}$ < 800 m/s. The C Class refers to deep and dense soil deposits with 180 m/s < $V_{s,30}$ < 360 m/s. Lastly, the D Class refers to low density soil deposits with $V_{s,30}$ < 180 m/s.

Quantitatively, 11/38 records belong to the A class, 20/38 belong to the B class and 7/38 belong to the C class. The little data coming from the D class was therefore not considered in this study.

Even if different processing procedures of the seismic signals may have a certain influence on the computed Newmark displacements (Crespellani et al., 2009), the record of the L’Aquila earthquake, the only belonging to the ITACA database, was taken into consideration however, because of its great interest for Italian seismicity.

All the ground motions are represented by means of two orthogonal horizontal acceleration time-histories and one vertical. In order to investigate the effect of the two horizontal components acting simultaneously, the records were coupled so as to maximize the effects. Considering that energy is proportional to the square of velocity, the direction along which the horizontal ground velocity became maximum ($PGV$) was researched. Subsequently, the two horizontal components of acceleration were summarised as vectors along this direction. Similar evaluations were carried out also summarising the component along the direction of the maximum $PGA$, founding that the approach which maximizes the instantaneous value of velocity generally gave the maximum kinematic effects on the sliding block in terms of displacements. The other kinematic parameters ($PGA$, $f^*$ and $t_{sig}$) reported in Table I were evaluated for the composed time-history of any single earthquake.

The selected 38 earthquakes were significant from an engineering point of view, as they represent all the allowable ground motions for which the PGA of the composed accelerograms was greater than 0.15g. The remaining records, giving a PGA lower than 0.15g, were not relevant to the purposes of this research.

The effective time of loading ($t_{sig}$) was evaluated in terms of “bracketed duration” [Bolt, 1969; Kramer, 1996], i.e. the elapsed time between the first and the last acceleration crossing through the ± 0.05g level in the time-history. The effective time of loading allows the seismic signal to be cleaned of components of very low amplitude, thus saving the remarkable acceleration history having an inertial effect on the masses.

For the approach proposed in this study, another basic parameter was required: the reference frequency of the earthquake. The Fourier analysis generally gives a range of prevalent frequencies, more or less wide, thus introducing a certain degree of subjectivity in the parameter.

To overcome this aspect, the mean frequency may be employed [Rathje et al., 1998], which averages the Fourier amplitude spectrum, weighting each period by the square of the corresponding Fourier amplitude according to the following expression:

$$f_{\text{mean}} = \frac{1}{T_{\text{mean}}} = \frac{\sum_i C_i^2}{\sum_i C_i^2 \left| f_i \right|}$$  

(1)

calculated in the range of frequencies $0.25 \text{ Hz} \leq f_i \leq 20 \text{ Hz}$.

In this expression, $C_i$ is the Fourier amplitude, associated with the frequency $f_i$, which is defined as the square root of the sum of the squares of the real and imaginary parts of the Fourier coefficient. Therefore, the mean frequency, being researched in the complete time history, involves all the frequencies of engineering interest.

However, this approach still requires a preliminary Fourier analysis. In a simplified framework approach, it has been thought to be more convenient to correlate the reference frequency with the kinematic parameters of the seismic record, by means of the following expression:

$$f^* = \frac{\omega}{2\pi} \equiv \left( \frac{PGA}{PGV} \right) \left( \frac{1}{2\pi} \right) \text{ (Hz)}$$  

(2)

For a sinusoidal variation of the ground motion, Expression (2) represents the fundamental fre-
frequency of the motion; for a generic seismic motion, it may represent a reference frequency which depends only on the ratio of two known kinematic quantities, PGA and PGV. Even if the reference frequency may be considered poorly representative of the real frequency content of the ground motion, it was used in this paper for normalization purposes, as it is easy to evaluate without the need to carry out further analysis. A comparison of the mean frequency, given by Fourier analysis (Eq. 1), and the reference frequencies, given by the Expression (2), is shown in Figure 1 for the composed accelerograms. A good agreement between the two approaches was observed, as shown by the grey area marking the 90% of data, especially for frequencies of less than 3 Hz (long period earthquakes). The coefficient of correlation, between the two variables, was $R^2=0.82$.

Preliminary Fourier analyses have also shown that the frequency content of the composed horizontal accelerogram does not basically change in re-

<table>
<thead>
<tr>
<th>Earthquake/ Date/ Hour</th>
<th>Recording Station</th>
<th>Site Class</th>
<th>Distance from Epicentre (km)</th>
<th>PGA (g)</th>
<th>PGV (m/s)</th>
<th>$f^*$ (Hz)</th>
<th>$t_{sig}$ (s)</th>
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<td>0.15</td>
<td>2.25</td>
<td>3.27</td>
</tr>
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</table>

(*) Supplied by ITACA database.
respect to NS and WE components. As shown in Figure 2a, for the whole set of records, the mean periods ($T_{\text{mean}} = 1/f_{\text{mean}}$) of the two components ($T_{\text{mean,NS}}$, $T_{\text{mean,WE}}$), and that of the composed record ($T_{\text{mean}}$), both coming from equation (1), are quite equivalent.

Similarly, analyses were carried out to verify the level of variation in using the composed accelerogram compared to the two composing records. By calculating the peak to peak displacements, related to the single ($S_{\text{NS}}, S_{\text{WE}}$) component and to the composed accelerograms ($S_{\text{comp}}$) (Fig. 2b), it was concluded that the composed accelerogram generally gives displacements greater than those associated with the single components. For this reason it was thought reasonable to work with the composed accelerograms, in order to maximize the effects of ground motion on the sliding Newmark’s block.

**Numerical model of sliding newmark’s block**

The numerical model employed in this work considers Newmark’s block resting on an inclined plane, and subjected to simultaneous horizontal and vertical ground movements. Columbian friction is considered, and no separation between the block and the plane is allowed.

Following this approach, the block remains in static equilibrium with the mobilized shear forces at its base, until the ground acceleration is lower than a critical value. When the ground acceleration becomes equal to the critical value, the mobilized shear force attains the limit friction. As soon as ground acceleration exceeds the critical value, the block starts to slide, thus achieving a new dynamic equilibrium. When ground acceleration decreases below the critical value, the block slows down and stops, until the ground acceleration exceeds the critical value again. As a result of this mechanism, the block accumulates displacements over the time.

If the ground motion occurs in the horizontal direction, two values of the critical acceleration can exist: one for the block motion activating downwards and the other for the motion activating up-
wards. Calling $\varphi$ the angle of friction of the block-plane interface and $\beta$ the angle between the plane and the horizontal direction, the critical acceleration for the activating motion downwards is as follows:

$$a_{\text{crit}} = \tan(\varphi - \beta)$$

(3)

The second critical acceleration could be of interest for low values of $\varphi$ and $\beta$; in this case upwards displacements are also possible:

$$a_{\text{crit,up}} = \frac{\tan \varphi + \tan \beta}{1 + \tan \varphi \tan \beta}$$

(4)

Generally speaking, the movement of a rigid block is described by means of the three equilibrium equations. However, if separation between the block and the plane is neglected, the motion is described only by only one differential equation, in which the known term takes into account the horizontal and the vertical components of the ground motion.

Referring to the symbolism of Figure 3, in a static condition the weight of the block ($W = mg$) can be decomposed in two components, one parallel to the plane ($T$) and the other normal ($N$).

The parallel component is equilibrated by the mobilised friction at the block base ($R$), whose maximum value is given by $R_{\text{max}} = \pm N \tan \varphi$ where the plus sign accounts for the downwards displacements of the block and the minus sign for the upwards displacements.

In dynamic conditions, the block can slide relatively to the plane in agreement with the following equation of dynamic equilibrium:

$$T - R_{\text{max}} = m \ddot{x}_{\text{tot}}$$

(5)

The term $\ddot{x}_{\text{tot}}$ accounts for the total acceleration of the block, evaluated in respect of an absolute system oriented in the same direction as the sloped plane. This acceleration may be expressed as the sum of the acceleration of the plane ($\ddot{y}_g$) and the relative acceleration ($\ddot{x}$) between the plane and the block.

Lastly, under dynamic conditions the mobilised friction ($R$), between the block and the plane is affected by the orthogonal component of the acceleration ($\ddot{y}_g$), which works by increasing or decreasing the normal load $N_{\text{dyn}} = N + m \ddot{y}_g = m g \cos \beta + m \ddot{y}_g$ and thus the contact force $R_{\text{max}} = \pm N_{\text{dyn}} \tan \varphi$. The expression (5) may now be rewritten in the following form:

$$\ddot{x} = g \sin \beta \pm g \cos \beta \tan \varphi \pm \ddot{y}_g \tan \varphi - \ddot{x}_g$$

(6)

The differential Equation (6) gives the time-history of the relative displacement of the block subjected to any known time-history of the ground motion, in terms of $x$-$y$ components. This equation does not depend on block mass, but only on the plane inclination and interface friction. Relative displacements are thus defined once initial conditions on displacement $x_r(t=0)$ and on velocity $\dot{x}_r(t=0)$ have been fixed.

In order to integrate the equations of motion, a linear variation of ground acceleration was supposed, inside a single step of time $\Delta t$.

Starting from at rest condition, the downward displacements activate when the action becomes greater than limit friction:

$$\ddot{x}_r < g \sin \beta - g \cos \beta \tan \varphi + \ddot{y}_g \tan \varphi - \ddot{x}_g$$

(7)

From this instant, relative displacement and velocity are positive ($x_r > 0$; $\dot{x}_r > 0$) and Equation (6) can be integrated until the motion stops ($\dot{x}_r = 0$).

Likewise, the upward displacements activate when the actions become greater than limit friction:

$$\ddot{x}_r > g \sin \beta - g \cos \beta \tan \varphi + \ddot{y}_g \tan \varphi - \ddot{x}_g$$

(8)

From this instant, relative displacement and velocity are negative ($x_r < 0$; $\dot{x}_r < 0$) and Equation (6) can be integrated until the motion stops ($\dot{x}_r = 0$).

Finally, the input accelerations $\ddot{x}_g$ and $\ddot{y}_g$, applied at the sloped plane can be expressed in terms of conventional horizontal $\dot{X}_g$ and vertical $\dot{Y}_g$ records in the absolute reference system $X$-$Y$ (Fig. 3).

When the ground acceleration exceeds the critical acceleration at the end of a step of time $\Delta t$, the resolutive algorithm researches the exact time at which the relative displacement starts, as also suggested by CRESPELLANI et al. [1990].
The numerical model has been developed with Matlab; integrations were performed using the time stepping of the selected records, or equal to 0.005s in the case of sinusoidal loading, independently from the frequency of the sinusoid.

A validation of the numerical code was carried out with regard to the closed form solutions given by Conte e Dente [1989] for horizontal impulsive loadings, rectangular, triangular and half wave of sinusoid. The coincidence between analytical and numerical solutions was verified.

In the literature, the analysis of the seismic displacements of the Newmark’s block has made extensive use of the critical acceleration together to some other kinematic parameters of the ground motion, always founding certain dispersion whenever a large database of seismic records was considered. The aim of this research was that of revising the physics of the phenomenon by distinguishing the behaviour of the block on the horizontal plane from that on the inclined plane, and by introducing also the effect of the vertical component of the acceleration. In fact, depending on the block configuration, some changes in the general trend may occur. For example, in the case of a regular cyclic base motion on the horizontal plane, the maximum displacement does not evolve with time, being linked only to the peak of the base motion; moreover, if friction is zero, the block displacement matches the peak to peak difference of the time history of the base displacement. Instead, the displacements on the inclined plane are linked to the succession of peaks exceeding the critical acceleration; therefore, the cumulated displacement is obviously connected to a duration parameter of the earthquake. In this latter case, if the interface friction angle is close to the slope angle, very large displacements may take place. Also, the direction in which the earthquake starts can introduce some appraisable differences in the displacements.

For the above-mentioned reasons, it was considered necessary to fully analyse the kinematics of the sliding block under wide conditions, in order to identify the source of any scattering unrelated to the variability of the seismic database used in the analysis.

Inclined plane: block displacements caused by horizontal seismic base motion

To evaluate the final displacements of a sliding block, numerical analyses were performed for the collected records, composed along the direction of maximum PGV. Various plane inclinations ($1° \leq \beta \leq 25°$) and friction angles ($11° \leq \phi \leq 35°$) were selected according to the common values involved by the geotechnical structures.

For all the couples of parameters $\beta$ and $\phi$, the maximum displacement was evaluated by applying the horizontal time histories of accelerations in both the positive and the negative verse. For example, Figure 4 shows the displacements of a block subjected to the L’Aquila earthquake, recorded at L’Aquila-V. Aterno on 6th April 2009 at 1.32 AM, in the case of $\beta = 15°$ and $\phi = 17°$ for both the directions of loading; due to the non-symmetry of the seismic signal, the direction of loading was very influential on the final displacements.

Referring to the whole seismic database of the selected Italian earthquakes, Figure 5 reports the
block displacements for $\beta =10^\circ$ and $\phi =11^\circ\pm 35^\circ$, in terms of the ratio $a_{\text{crit}}/\text{PGA}$. It is possible to highlight a sensible dispersion of data, which increases as the ratio $a_{\text{crit}}/\text{PGA}$ decreases, however remaining limited to about two orders of magnitude irrespective of the soil conditions of the recording station. Many authors have investigated the possibility of correlating the displacements with further kinematic parameters in order to reduce the dispersion of data [AMBRASEYS e Menu, 1988; Whitman e Liao, 1984; YEGIAN et al., 1991; JIRSON, 1993; Crespellani et al., 1998; Cai and Bathurst, 1996] and a synthesis of these correlations was recently reported for the Italian earthquakes by some authors [ROMEO, 2000; AUSILIO et al., 2007; MADIAI, 2009].

Based on the previous work of YEGIAN et al. (1991), it was thought to be attractive to explore the possibility of a new approach based on the equivalence with a sinusoidal base motion. YEGIAN et al. (1991) predicted the normalised permanent displacement ($S_n$) by means of the following polynomial function:

$$\log S_n=0.22-10.12\left[\frac{a_{\text{crit}}}{\text{PGA}}\right]+16.38\left[\frac{a_{\text{crit}}}{\text{PGA}}\right]^2-11.48\left[\frac{a_{\text{crit}}}{\text{PGA}}\right]^3$$

(9)

being:

$$S_n=\frac{S}{\text{PGA} N_{eq} T^2}$$

(10)

in which $S$ is the permanent displacement and $T$ is the predominant period of the ground motion. The parameter $N_{eq}$ represents the number of equivalent cycles of a given earthquake computed on the basis of the Seed et al. [1983] approach for the risk of soil liquefaction.

In this work, a physical equivalence was established between seismic and sinusoidal loadings to evaluate $N_{eq}$ for a sliding block, thus obtaining a predictive relationship fully based on the kinematic properties of a given ground motion.

Inclined plane: displacements caused by horizontal sinusoidal base motion

Analyses were carried out to characterise the dynamic response of a sliding block subjected to a generic sinusoidal base motion $\ddot{X}_e=a_0\text{sen}(2\pi ft)$. Various combinations were examined: the sloping angle $\beta$ was varied in a range of between $1^\circ$ and $25^\circ$; the friction angle $\phi$ between $11^\circ$ and $35^\circ$; the frequency $f$ between $1Hz$ and $5Hz$, and the acceleration amplitude $a_0$ between $0.1g$ and $1.0g$.

Under steady-state base motion, the mobilised block displacements is constant on every cycle of loading (Fig. 6), therefore, the cumulated displacement is directly proportional to the duration of the loading. An average gradient of displacement ($\Delta s/\Delta t$) was introduced for comparison purposes; moreover, the frequency $f$ of the base motion also affects the block displacements: as frequency increases, displacement decreases in a linear proportion.

On the basis of the previous findings, it was proved that the results can be normalised with respect to both time and frequency by introducing a non-dimensional parameter:

$$\frac{\Delta s/\delta}{\Delta t a_0}$$

(11)

being:

$$\delta=\frac{\cos\phi}{\cos(\phi-\beta)}$$

(12)

By doing so, the results can be generalised to any given horizontal sinusoidal base motion.

A synthesis of the normalised block displacements, obtained by numerical analyses, is shown in Figure 7 for different slope angles and different values of the ratio $a_{\text{crit}}/a_0$. A little dispersion of data is observed for the minimum slope angle $\beta=5^\circ$; this happens because the amplitude of the sinusoidal acceleration may exceed the critical upward acceleration ($a_{\text{crit},\text{up}}$), therefore $\Delta s/\Delta t$ also depends on $a_{\text{crit},\text{up}}$. This dispersion rapidly increases as the slope angle becomes close to zero, because $\Delta s/\Delta t$ approaches zero. Apart from the cases in which $\beta<5^\circ$, the whole displacement data can be described by the following relationship:

$$\frac{\Delta s}{\Delta t} = \frac{1}{f} \frac{a_0}{\delta} \left| \frac{a_{\text{crit}}}{a_0} \right| G$$

(13)
Inclined plane: influence of vertical ground motion on block displacements

In order to evaluate the influence of vertical ground motion on the final seismic displacements of a block, analyses were carried out for a sloped plane varying in inclination between 5° and 25°, and a friction angle varying between 8° and 38°. In the following, the results related to the events of Irpinia (1980) and L’Aquila (2009), are presented. These earthquakes are characterised by an appreciable vertical component of acceleration ($0.235g < PGA_{V} < 0.522g$ and $0.71 < PGA_{V}/PGA_{H} < 0.81$).

The results are shown in terms of cumulated block displacements induced by the horizontal component alone, and by horizontal and vertical components both acting simultaneously. In this latter case, both positive (+) and negative (−) verses of the vertical component of acceleration have been considered. The results for Irpinia and L’Aquila earthquakes are shown in Figures 8a and 8b, respectively.

As can be expected, the maximum final displacement $s$ is inversely proportional to friction interface $\phi$ and directly proportional to the slope angle $\beta$; for friction angles approaching the slope angles, the resulting displacement tends to be unlimited. For the two time histories, the influence of the vertical accelerations on the final block displacement is negligible, and the small differences tend to disappear for large displacements. A similar behaviour was also found by Simonelli and Di Stefano [2001].

Many factors may explain these results, which seems typical of many seismic events: for example the random nature of the seismic signal, that does not involve a substantial synchronism of the peaks of acceleration, along both the vertical and the horizontal directions.

A very different situation arises in the case of sinusoidal horizontal and vertical base motions acting simultaneously. Being the duration of a sinusoidal loading unlimited, a comparison was carried out in terms of gradient of displacement in time. The results, shown in Figure 9, refer to a 15° sloped plane, subjected to horizontal and vertical sinusoidal loadings with different amplitude, frequency and phase ratios. The reference horizontal component was maintained constant, with amplitude $a_{0,H}=0.6g$ and frequency $f_{H}=2$ Hz.

Figure 9a shows the results of the cases in which the ratio, between vertical and horizontal amplitude of loading, was changed in the range of between 0 and 1, being frequency and phase equal for both loadings.

Figure 9b refers to the case of in phase components, in which the frequency of the vertical component ($f_{V}$) was changed within the range of $0.5 f_{H}$ and $2 f_{H}$ and the ratio of amplitudes was fixed at 0.5.

Finally, Figure 9c refers to the case in which the amplitude and frequency of the vertical loading were assumed to be equal to that of horizontal loading, while the phase difference ($\Delta \text{Phase}$) was varied in the range of 0° to 180°.

Looking at Figure 9, it can be deduced that the dynamic response of the block is greatly affected by all the above mentioned parameters of the motion, especially for the greatest friction angles. For a lower friction angle, the influence of the vertical component is practically negligible, as is also observed in the case of seismic events (Fig. 8).

On the basis of the experience gained for seismic Italian events, it has been thought that an equivalence criterion between seismic and sinusoidal loadings could be referred only to horizontal components.

The equivalence criterion

The maximum displacements induced by the Italian earthquakes were reproduced by horizontal sinusoidal base motions. The significant time and the reference frequency were used to establish an equivalent criterion between the cumulated displacements induced by a seismic event and those...
coming from an equivalent sinusoidal ground motion. The duration of the sinusoid was assumed to be coincident with the bracketed duration of the seismic event, while the frequency was considered to be equal to the reference frequency of the earthquake. The amplitude of the equivalent sinusoidal loading was appraised by a recursive analysis, until the corresponding seismic displacement was supplied.

The results of the equivalence analysis are shown in Figure 10, in which the equivalent acceleration ($a_{eq}$) is related to the $PGA$ of the earthquake and to the critical acceleration of the block ($a_{crit}$). For a given $PGA$, the equivalent acceleration is solely dependent on critical acceleration. This means that different couples of $\phi$ and $\beta$, under the condition $\phi - \beta = constant$, give the same equivalent acceleration. It is possible to observe that, for a given critical acceleration, the relationship between $a_{eq}$ and $PGA$ is almost linear, with a moderate dispersion decreasing as $(\phi - \beta)$ increases. Moreover, the existence of a threshold value is clearly highlighted in the same Figure 10; in fact, relative displacement, between the block and the inclined plane, did not occur if acceleration was less than critical acceleration.

Another consideration is that the interpolating line of numerical results does not show any dependence from soil type; this is due to the fact that the kinematic parameters of ground motions considered here, already take into account the soil-induced modification in the earthquake time history.

A filter-effect was highlighted by analyses: as friction decreases, dispersion increases, due to the frequency content in the ground motion; therefore, a considerable amount of friction is able to filter the low amplitude components of the frequency spectrum.
The synthesis of the numerical simulations is proposed in terms of normalised accelerations (Fig. 11); in this representation the ratio $a_{\text{crit}} / a_{\text{eq}}$ is related to the ratio $a_{\text{crit}} / \text{PGA}$. The data defines the following correlation with moderate dispersion:

$$
\frac{a_{\text{crit}}}{a_{\text{eq}}} = 1.1 \left[ 1 - \frac{1}{1 + 10 \left( a_{\text{crit}} / \text{PGA} \right)^{1.4}} \right] \quad (15)
$$

As expected, when the ratio $a_{\text{crit}} / \text{PGA}$ becomes close to zero or to unit, the ratio $a_{\text{crit}} / a_{\text{eq}}$ also attains the same values.

For comparison, the Equation (15) is also shown in Figure 10 as relationship between $a_{\text{eq}}$ and $\text{PGA}$, for varying values of $a_{\text{crit}}$.

Apart from some unavoidable dispersion, the mean trend of Figure 11 allows the equivalent acceleration of a generic Italian earthquake to be estimated on the basis of $\text{PGA}$ alone. Thereafter, by using Figure 7 or Equation (13), it is also possible to evaluate the post seismic block displacement, recalling that $a_0 = a_{\text{eq}}$ and being $t_{\text{eq}}$ and $\text{PGA}/\text{PGV}$ already known.

A comparison between the seismic displacements computed by numerical analysis and those expected by using the equivalence approach is shown in Figure 12. The dispersion of data, not exceeding one order of magnitude, is uniform in the range of displacements of engineering interest.

The level of sensitivity of the approach followed in this analysis was compared with the more consolidated WHITMAN and LIAO [1984] approach (Fig. 13). These latter authors predicted the permanent displacement over inclined plane by means of the following expression:

$$
\delta = A \left( \frac{\text{PGV}^2}{\text{PGA}} \right) \left( \frac{a_{\text{eq}}}{\text{PGA}} \right) \left( \frac{\text{PGA}}{\text{PGV}} \right) \quad (16)
$$

in which $A = 37$ and $B = -9.4$ are two constants, formerly evaluated by the authors via regression analysis, for earthquakes of magnitude varying between 6.3 and 6.7, and here verified to be fine also for the whole selected Italian database, with magnitude varying between 3.7 and 6.5.

By comparing Figures 12 and 13, it can be deduced that the expression of WHITMAN and LIAO [1984] gave a level of dispersion comparable with that obtained by using the equivalent approach of this paper. Even if the strong motion duration was considered an influent parameter in seismic engineering [RATHJE et al., 1998; BOMMER and MARTINEZ-
Pereira, 1999], in the approach outlined in this paper, the introduction of a parameter of earthquake duration (\(t_{sig}\)) was not able to enhance the results. For example, Figure 14(a,b) shows that similar results were achieved by applying the equivalent approach with other definitions of the earthquake duration, such as the \textit{uniform duration} [Bolt, 1973] and the \textit{significant duration} [Trifunac and Brady, 1975]. As a consequence, future improvements of the correlations should investigate more representative kinematic parameters of the earthquake.

Seismic block displacements on a horizontal plane

When the plane is horizontal it is not possible to apply the equivalent approach, as previously defined for the inclined plane. In fact, under a sinusoidal loading the block oscillates between two opposite positions, constant in time, so that the displacement gradient becomes zero. The absence of a gradient of displacement does not allow the evaluation of the equivalent acceleration of the reference sinusoid, acting for a significant time \(t_{sig}\).

Some previous authors [Newmark, 1965; Richards and Elms, 1979; Whitman and Liao, 1984], have shown that a good correlation can be referred to \(PGV\), \(PGA\) and \(a_{eq}\), even if they were referring to structures under asymmetric conditions of loading, such as gravity walls.

A similar approach has been followed in this work for the Italian database under the hypothesis of symmetric frictional strength of the block. The maximum peak to peak displacement \(s\) may be expressed in the following form:

\[
s = A \frac{PGV^2}{PGA} \left(1 - \frac{a_{eq}}{PGA}\right)^B
\]  

(17)

\(A\) and \(B\) being two constants, coming from regression analysis.

On the basis of the numerical analyses, carried out for the Italian earthquake database, the best fitting values are: \(A = 5\) and \(B = 3\) (Fig. 15) with a coefficient of correlation \(R^2=0.79\).

As in inclined planes, the correlation is not dependent on soil type, confirming that, in the sliding block case, \(PGV\) and \(PGA\) automatically account for the nature of the site.
Moreover, it is interesting to highlight that as $acrit$ approaches zero, the relative block displacement approaches the maximum peak-to-peak ground displacement.

Finally, adapting the Expression (16) of Whitman and Liao [1984] to the block on horizontal plane, subjected to the Italian database, the values $A = 17$ and $B = -8$ were obtained. As shown in Figure 15, the resulting correlation indicated less agreement with the numerical results in respect to the case of inclined plane, with a coefficient of correlation $R^2=0.66$.

Conclusion

The aim of this work was to investigate the kinematics of a Newmark block, with the aim of improving the predictions. The subject founds its interest in the quick evaluation of displacements of a wide class of geotechnical structures, spacing from the quiescent landslides to earth dams and gravity works. For this purpose, a simplified procedure was calibrated, based on the equivalent sinusoidal ground motion, capable of giving the same displacements as seismic ones.

Three kinematic parameters of a ground motion were considered: the peak ground acceleration ($PGA$), the peak ground velocity ($PGV$) and the bracketed duration ($t_{sig}$). Instead, the critical acceleration ($acrit$) is related only to the interface friction and to the slope angle.

Starting from the SISMA and ITACA databases of Italian earthquakes, numerical integrations of the equation of the block motion were performed to obtain a wide range of post-seismic displacements for various frictions and sloping angles. The dynamic response analyses were also carried out for a sinusoidal base motion, of generic frequency, amplitude and duration.

By comparing the effects, an equivalence criterion was established for the two types of loading. This approach provides the same seismic displacements by using sinusoidal base motion, having the same reference frequency and the same bracketed duration ($t_{sig}$) of the earthquake. The third parameter, the amplitude of the equivalent sinusoid, was obtained by fitting the displacements of the block. Thereafter, the sinusoidal ground motion was used to compare the effects of a number of irregular seismic motions.

The results of the analyses indicate a good correlation (Fig. 10) between the equivalent acceleration ($aeq$) and the peak ground acceleration ($PGA$). Moreover, for a given $PGA$, $aeq$ is solely dependent on $acrit$, as is highlighted in Figure 11.

On the basis of the $PGA$, it is possible to estimate the equivalent acceleration from Figure 11 or Equation (15) for any level of $acrit$. Knowing the $t_{sig}$ and $PGA/PGV$, it is possible to evaluate the post-seismic block displacement by using Figure 7 or Equation (13). Following this approach, it was possible to estimate the displacement within one order of magnitude of uncertainty (Fig. 12).

However, the proposed approach gave a level of dispersion comparable with that associated with the best correlations found in literature [Whitman and Liao, 1984; Madai, 2009], and based only on two parameters, PGA and PGV. Even if the strong motion duration was considered an influent parameter in seismic engineering [Rathje et al., 1998; Bommer and Martinez-Pereira, 1999], in the approach outlined in this paper, the introduction of a parameter of earthquake duration ($t_{sig}$) was not able to enhance the results, even considering other definitions of the earthquake duration, such as the uniform duration [Bolt, 1973] and the significant duration [Trifunac and Brady, 1975].

As consequence, future improvements of the correlations should investigate kinematic parameters which are able to describe the effects of an earthquake in a more representative way.

In the case of a block on a horizontal plane, the previous procedure is not applicable; however, numerical analysis carried out with the same database, confirms that post-seismic displacements may be suitably correlated only to $PGA$ and $PGV$ by a simple relationship (Fig. 15 or Eq. 17).

Finally, the results from Italian earthquakes, on both horizontal and inclined planes, show that the displacements of a Newmark’s block are not influenced by the vertical component of ground motion. Moreover, by using the abovementioned kinematic parameters, the displacements are not influenced by the nature of the soil beneath the recording station.
References

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Sollecitazione equivalente per lo studio della risposta sismica del blocco di Newmark

Sommario

Partendo da una raccolta di registrazioni di eventi sismici Italiani, provenienti dalle banche dati SISMA e ITACA, si è tentato di valutare gli spostamenti sismici per un blocco Newmark provvisto di attrito Coulombiano e disposto sia su piano inclinato che orizzontale. Tale argomento interessa la progettazione sismica di strutture geotecniche in grado di accumulare spostamenti, quali le frane quiescenti, le dighe in terra e le opere a gravità.

In relazione all’inclinazione del piano e all’attrito d’interfaccia, si sono condotte svariate integrazioni numeriche delle equazioni del moto, al fine di valutare gli spostamenti finali del blocco sottoposto a 38 eventi sismici, di magnitudo compresa tra 3.7 < M< 6.5.

Le analisi sono state eseguite sia nell’ipotesi di sola accelerazione orizzontale agente alla base del modello che nel caso di accelerazioni simultanee, orizzontale e verticale.

Gli spostamenti del blocco sono stati valutati anche per moti sinusoidali, variabili in ampiezza, durata e frequenza. Tali spostamenti sono stati utilizzati per un processo di

normalizzazione: confrontando gli spostamenti indotti dai sismi con quelli generati da forzanti sinusoidali, in un intervallo di tempo pari alla bracketed duration del sisma (tsg), si è pervenuti a un criterio di equivalenza che fornisce l’accelerazione equivalente (a<sub>eq</sub>) di un moto sinusoidale, in grado di produrre gli stessi spostamenti del terremoto di progetto.

I risultati delle analisi indicano una buona correlazione (fig. 10) tra l’accelerazione equivalente (a<sub>eq</sub>) e l’accelerazione di picco (PGA) di un sisma reale. Inoltre, per un dato PGA, a<sub>eq</sub> è dipendente unicamente dall’accelerazione critica (a<sub>cr</sub>) del blocco su piano inclinato, come evidenziato nella figura 11.

Sulla base del PGA, è possibile stimare l’accelerazione equivalente tramite la figura 11 o l’equazione (15), per qualsiasi livello di a<sub>cr</sub>, noti che siano tsg e PGA/PGV, è possibile valutare lo spostamento sismico del blocco, utilizzando la figura 7 o l’equazione (13).

Nonostante l’approccio proposto si basi su una precisa corrispondenza fisica tra l’effetto del moto sismico e quello di un moto ciclico regolare, si è osservato che il livello di dispersione dei dati, con l’approccio proposto, è dello stesso ordine di grandezza di quello mostrato dalle migliori correlazioni presenti nella letteratura tecnica, basate solo sui due parametri cinematici PGA e PGV [WHITMAN and LIAO 1984; MADIAI, 2009]. In definitiva, l’introduzione di un ulteriore parametro cinematico, relativo alla durata significativa del terremoto (t<sub>sig</sub>), non apporta alcun miglioramento sulle previsioni di spostamento. Ad analoghe conclusioni si perviene anche ricorrendo ad altre definizioni della durata della fase strong-motion, quali la uniform duration [BOLT, 1973] e la significant duration [TRIFUNAC and BRADY, 1975].

Se ne deduce che in futuro occorrerà rivolgere l’attenzione allo studio di parametri sismici che, relativamente alla cinematica del blocco ad attrito, siano più rappresentativi degli effetti di un terremoto.

Nel caso di blocco su piano orizzontale, la procedura equivalente non può essere applicata non potendo definire un gradiente di spostamento sotto una forzante sinusoidale regolare. Tuttavia, le analisi numeriche eseguite con il medesimo database confermano, anche in questo caso, come gli spostamenti possano essere previsti facendo riferimento solo ai due parametri cinematici PGA e PGV (Fig. 15 o Eq.17); in tal caso, la dispersione è dello stesso ordine di grandezza di quella riscontrata per il blocco su piano inclinato.

Infine, i risultati delle analisi, sia su piano inclinato che orizzontale, mostrano come gli spostamenti di un blocco di Newmark non siano influenzati dalla componente verticale del moto del suolo. Inoltre, utilizzando i parametri cinematici summenzionati, gli spostamenti non mostrano alcuna dipendenza dalla natura del terreno di fondazione.